

ANÁLISIS MATEMÁTICO II. Integral Indefinida

I. Calcular las siguientes integrales:

1. $\int x^5 dx$. Resp.: $\frac{x^6}{6} + C$.
2. $\int (x + \sqrt{x}) dx$. Resp.: $\frac{x^2}{2} + \frac{2x\sqrt{x}}{3} + C$.
3. $\int \left(\frac{3}{\sqrt{x}} - \frac{x\sqrt{x}}{4} \right) dx$. Resp.: $6\sqrt{x} - \frac{1}{10}x^2\sqrt{x} + C$.
4. $\int \frac{x^2}{\sqrt{x}} dx$. Resp.: $\frac{2}{5}x^2\sqrt{x} + C$.
5. $\int \left(\frac{1}{x^2} - \frac{4}{x\sqrt{x}} + 2 \right) dx$. Resp.: $-\frac{1}{x} - \frac{8}{\sqrt{x}} - 2x + C$.
6. $\int \frac{dx}{\sqrt{x}}$. Resp.: $2\sqrt{x} + C$.
7. $\int \left(x^2 + \frac{1}{\sqrt[3]{x}} \right)^2 dx$. Resp.: $\frac{x^5}{5} + \frac{3}{4}x^2\sqrt[3]{x^2} + 3\sqrt[3]{x} + C$.

Integración por sustitución

8. $\int e^{5x} dx$. Resp.: $\frac{1}{5}e^{5x} + C$.
9. $\int \cos 5x dx$. Resp.: $\frac{\operatorname{sen} 5x}{5} + C$.
10. $\int \operatorname{sen} ax dx$. Resp.: $\frac{-\cos ax}{a} + C$.
11. $\int \frac{\ln x}{x} dx$. Resp.: $\frac{1}{2}\ln^2 x + C$.
12. $\int \frac{dx}{\operatorname{sen}^2 3x}$. Resp.: $\frac{-\cot g 3x}{3} + C$.
13. $\int \frac{dx}{\cos^2 7x}$. Resp.: $\frac{\operatorname{tg} 7x}{7} + C$.
14. $\int \frac{dx}{3x - 7}$. Resp.: $\frac{1}{3}\ln|3x - 7| + C$.

15. $\int \frac{dx}{1-x}$. Resp.: $-\ln|1-x|+C$.
16. $\int \frac{dx}{5-2x}$. Resp.: $-\frac{1}{2}\ln|5-2x|+C$.
17. $\int tg2x dx$. Resp.: $-\frac{1}{2}\ln|\cos 2x|+C$.
18. $\int \cot g(5x-7) dx$. Resp.: $\frac{1}{5}\ln|\operatorname{sen}(5x-7)|+C$.
19. $\int \frac{dy}{\cot g 3y}$. Resp.: $-\frac{1}{3}\ln|\cos 3y|+C$.
20. $\int \cot g \frac{x}{3} dx$. Resp.: $3\ln|\operatorname{sen} \frac{x}{3}|+C$.
21. $\int tg x \cdot \sec^2 x dx$. Resp.: $\frac{1}{2}tg^2 x+C$.
22. $\int (ctg e^x)e^x dx$. Resp.: $\ln|\operatorname{sene}^x|+C$.
23. $\int \left(tg 4S - \cot g \frac{S}{4} \right) dS$. Resp.: $-\frac{1}{4}\ln|\cos 4S| - 4\ln|\operatorname{sen} \frac{S}{4}|+C$.
24. $\int \operatorname{sen}^2 x \cos x dx$. Resp.: $\frac{\operatorname{sen}^3 x}{3}+C$.
25. $\int \cos^3 x \operatorname{sen} x dx$. Resp.: $\frac{-\cos^4 x}{4}+C$.
26. $\int \sqrt{x^2+1} x dx$. Resp.: $\frac{1}{3}\sqrt{(x^2+1)^3}+C$.
27. $\int \frac{x dx}{\sqrt{2x^2+3}}$. Resp.: $\frac{1}{2}\sqrt{2x^2+3}+C$.
28. $\int \frac{x^2 dx}{\sqrt{x^3+1}}$. Resp.: $\frac{2}{3}\sqrt{x^3+1}+C$.
29. $\int \frac{\cos x dx}{\operatorname{sen}^2 x}$. Resp.: $\frac{-1}{\operatorname{sen} x}+C$.
30. $\int \frac{\operatorname{sen} x dx}{\cos^3 x}$. Resp.: $\frac{1}{2\cos^2 x}+C$.

31. $\int \frac{tgx}{\cos^2 x} dx$. Resp.: $\frac{tg^2 x}{2} + C$.
32. $\int \frac{\cot g x dx}{\sen^2 x}$. Resp.: $-\frac{\cot g^2 x}{2} + C$.
33. $\int \frac{dx}{\cos^2 x \sqrt{tgx - 1}}$. Resp.: $2\sqrt{tgx - 1} + C$.
34. $\int \frac{\ln(x+1)}{x+1} dx$. Resp.: $\frac{\ln^2(x+1)}{2} + C$.
35. $\int \frac{\cos x dx}{\sqrt{2\sen x + 1}}$. Resp.: $\sqrt{2\sen x + 1} + C$.
36. $\int \frac{\sen 2x dx}{(1 + \cos 2x)^2}$. Resp.: $\frac{1}{2(1 + \cos 2x)} + C$.
37. $\int \frac{\sen 2x dx}{\sqrt{1 + \sen^2 x}}$. Resp.: $2\sqrt{1 + \sen^2 x} + C$.
38. $\int \frac{\sqrt{tgx + 1}}{\cos^2 x} dx$. Resp.: $\frac{2}{3}\sqrt{(tgx + 1)^3} + C$.
39. $\int \frac{\cos 2x dx}{(2 + 3\sen 2x)^3}$. Resp.: $\frac{-1}{12} \frac{1}{(2 + 3\sen 2x)^2} + C$.
40. $\int \frac{\sen 3x dx}{\sqrt[3]{\cos^4 3x}}$. Resp.: $\frac{1}{\sqrt[3]{\cos 3x}} + C$.
41. $\int \frac{\ln^2 x dx}{x}$. Resp.: $\frac{\ln^3 x}{3} + C$.
42. $\int \frac{\arcsen x dx}{\sqrt{1 - x^2}}$. Resp.: $\frac{\arcsen^2 x}{2} + C$.
43. $\int \frac{\arctg x dx}{1 + x^2}$. Resp.: $\frac{\arctg^2 x}{2} + C$.
44. $\int \frac{\arccos^2 x}{\sqrt{1 - x^2}} dx$. Resp.: $-\frac{\arccos^3 x}{3} + C$.
45. $\int \frac{\operatorname{arccot} g x}{1 + x^2} dx$. Resp.: $-\frac{\operatorname{arccot} g^2 x}{2} + C$.

46. $\int \frac{x dx}{x^2 + 1}$. Resp.: $\frac{1}{2} \ln(x^2 + 1) + C$.
47. $\int \frac{x + 1}{x^2 + 2x + 3} dx$. Resp.: $\frac{1}{2} \ln(x^2 + 2x + 3) + C$.
48. $\int \frac{\cos x dx}{2 \sin x + 3}$. Resp.: $\frac{1}{2} \ln(2 \sin x + 3) + C$.
49. $\int \frac{dx}{x \ln x}$. Resp.: $\ln |\ln x| + C$.
50. $\int 2x(x^2 + 1)^4 dx$. Resp.: $\frac{(x^2 + 1)^5}{5} + C$.
51. $\int tg^4 x dx$. Resp.: $\frac{tg^3 x}{3} - tg x + x + C$.
52. $\int \frac{dx}{(1 + x^2) \arctg x}$. Resp.: $\ln |\arctg x| + C$.
53. $\int \frac{dx}{\cos^2 x (3tg x + 1)}$. Resp.: $\frac{1}{3} \ln |3tg x + 1| + C$.
54. $\int \frac{tg^3 x}{\cos^2 x} dx$. Resp.: $\frac{tg^4 x}{4} + C$.
55. $\int \frac{dx}{\sqrt{1 - x^2} \arcsen x}$. Resp.: $\ln |\arcsen x| + C$.
56. $\int \frac{\cos 2x}{2 + 3 \sin 2x} dx$. Resp.: $\frac{1}{6} \ln |2 + 3 \sin 2x| + C$.
57. $\int \cos(\ln x) \frac{dx}{x}$. Resp.: $\sin(\ln x) + C$.
58. $\int \cos(a + bx) dx$. Resp.: $\frac{1}{b} \sin(a + bx) + C$.
59. $\int e^{2x} dx$. Resp.: $\frac{1}{2} e^{2x} + C$.
60. $\int e^{\frac{x}{3}} dx$. Resp.: $3e^{\frac{x}{3}} + C$.
61. $\int e^{\sin x} \cos x dx$. Resp.: $e^{\sin x} + C$.

62. $\int a^{x^2} x dx$. Resp.: $\frac{a^{x^2}}{2 \ln a} + C$.
63. $\int e^{\frac{x}{a}} dx$. Resp.: $a e^{\frac{x}{a}} + C$.
64. $\int (e^{2x})^2 dx$. Resp.: $\frac{1}{4} e^{4x} + C$.
65. $\int 3^x e^x dx$. Resp.: $\frac{3^x e^x}{\ln 3 + 1} + C$.
66. $\int e^{-3x} dx$. Resp.: $-\frac{1}{3} e^{-3x} + C$.
67. $\int (e^{5x} + a^{5x}) dx$. Resp.: $\frac{1}{5} \left(e^{5x} + \frac{a^{5x}}{\ln a} + C \right)$.
68. $\int e^{x^2+4x+3} (x+2) dx$. Resp.: $\frac{1}{2} e^{x^2+4x+3} + C$.
69. $\int \frac{(a^x - b^x)^2}{a^x b^x} dx$. Resp.: $\frac{\left(\frac{a}{b}\right)^x - \left(\frac{b}{a}\right)^x}{\ln a - \ln b} - 2x + C$.
70. $\int \frac{e^x dx}{3 + 4e^x}$. Resp.: $\frac{1}{4} \ln(3 + 4e^x) + C$.
71. $\int \frac{e^{2x} dx}{2 + e^{2x}}$. Resp.: $\frac{1}{2} \ln(2 + e^{2x}) + C$.
72. $\int \frac{dx}{1 + 2x^2}$. Resp.: $\frac{1}{\sqrt{2}} \operatorname{arctg}(\sqrt{2}x) + C$.
73. $\int \frac{dx}{\sqrt{1 - 3x^2}}$. Resp.: $\frac{1}{\sqrt{3}} \operatorname{arcsen}(\sqrt{3}x) + C$.
74. $\int \frac{dx}{\sqrt{16 - 9x^2}}$. Resp.: $\frac{1}{3} \operatorname{arcsen} \frac{3x}{4} + C$.
75. $\int \frac{dx}{\sqrt{9 - x^2}}$. Resp.: $\operatorname{arcsen} \frac{x}{3} + C$.
76. $\int \frac{dx}{4 + x^2}$. Resp.: $\frac{1}{2} \operatorname{arctg} \frac{x}{2} + C$.

77. $\int \frac{dx}{9x^2 + 4}$. Resp.: $\frac{1}{6} \operatorname{arctg} \frac{3x}{2} + C$.
78. $\int \frac{dx}{4 - 9x^2}$. Resp.: $\frac{1}{12} \ln \left| \frac{2 + 3x}{2 - 3x} \right| + C$.
79. $\int \frac{dx}{\sqrt{x^2 + 9}}$. Resp.: $\ln|x + \sqrt{x^2 + 9}| + C$.
80. $\int \frac{dx}{\sqrt{b^2x^2 - a^2}}$. Resp.: $\frac{1}{b} \ln \left| bx + \sqrt{b^2x^2 - a^2} \right| + C$.
81. $\int \frac{dx}{\sqrt{b^2 + a^2x^2}}$. Resp.: $\frac{1}{a} \ln \left| ax + \sqrt{b^2 + a^2x^2} \right| + C$.
82. $\int \frac{dx}{a^2x^2 - e^2}$. Resp.: $\frac{1}{2ae} \ln \left| \frac{ax - e}{ax + e} \right| + C$.
83. $\int \frac{x^2 dx}{5 - x^6}$. Resp.: $\frac{1}{6\sqrt{5}} \ln \left| \frac{x^3 + \sqrt{5}}{x^3 - \sqrt{5}} \right| + C$.
84. $\int \frac{x dx}{\sqrt{1 - x^4}}$. Resp.: $\frac{1}{2} \operatorname{arcsen} x^2 + C$.
85. $\int \frac{x dx}{x^4 + a^4}$. Resp.: $\frac{1}{2a^2} \operatorname{arctg} \frac{x^2}{a^2} + C$.
86. $\int \frac{e^x dx}{\sqrt{1 - e^{2x}}}$. Resp.: $\operatorname{arcsen} e^x + C$.
87. $\int \frac{dx}{\sqrt{3 - 5x^2}}$. Resp.: $\frac{1}{\sqrt{5}} \operatorname{arcsen} \sqrt{\frac{5}{3}} x + C$.
88. $\int \frac{\cos x dx}{a^2 + \operatorname{sen}^2 x}$. Resp.: $\frac{1}{a} \operatorname{arctg} \left(\frac{\operatorname{sen} x}{a} \right) + C$.
89. $\int \frac{dx}{x\sqrt{1 - \ln^2 x}}$. Resp.: $\operatorname{arcsen}(\ln x) + C$.
90. $\int \frac{\arccos x - x}{\sqrt{1 - x^2}} dx$. Resp.: $\frac{-1}{2} (\arccos x)^2 + \sqrt{1 - x^2} + C$.
91. $\int \frac{x - \operatorname{arctg} x}{1 + x^2} dx$. Resp.: $\frac{1}{2} \ln(1 + x^2) - \frac{1}{2} (\operatorname{arctg} x)^2 + C$.

$$92. \int \frac{\sqrt{1+\ln x}}{x} dx. \text{ Resp.: } \frac{2}{3}\sqrt{(1+\ln x)^3}+C.$$

$$93. \int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} dx. \text{ Resp.: } \frac{4}{3}\sqrt{(1+\sqrt{x})^3}+C.$$

$$94. \int \frac{dx}{\sqrt{x}\sqrt{1+\sqrt{x}}}. \text{ Resp.: } 4\sqrt{1+\sqrt{x}}+C.$$

$$95. \int \frac{e^x dx}{1+e^{2x}}. \text{ Resp.: } \operatorname{arctg} e^x + C.$$

$$96. \int \frac{\cos x dx}{\sqrt[3]{\sin^2 x}}. \text{ Resp.: } 3\sqrt[3]{\sin x} + C.$$

$$97. \int \sqrt{1+3\cos^2 x} \sin 2x dx. \text{ Resp.: } \frac{-2}{9}\sqrt{(1+3\cos^2 x)^3} + C.$$

$$98. \int \frac{\sin 2x}{\sqrt{1+\cos^2 x}} dx. \text{ Resp.: } -2\sqrt{1+\cos^2 x} + C.$$

$$99. \int \frac{\cos^3 x}{\sin^4 x} dx. \text{ Resp.: } \frac{1}{\sin x} - \frac{1}{3\sin^3 x} + C.$$

$$100. \int \frac{\sqrt[3]{\tan^2 x}}{\cos^2 x} dx. \text{ Resp.: } \frac{3}{5}\sqrt[3]{\tan^5 x} + C.$$

$$101. \int \frac{dx}{2\sin^2 x + 3\cos^2 x}. \text{ Resp.: } \frac{1}{\sqrt{6}} \operatorname{arctg} \left(\sqrt{\frac{2}{3}} \tan x \right) + C.$$

Integrales del tipo $\int \frac{Ax+B}{ax^2+bx+c} dx$:

$$102. \int \frac{dx}{x^2+2x+5}. \text{ Resp.: } \frac{1}{2} \operatorname{arctg} \frac{x+1}{2} + C.$$

$$103. \int \frac{dx}{3x^2-2x+4}. \text{ Resp.: } \frac{1}{\sqrt{11}} \operatorname{arctg} \frac{3x-1}{\sqrt{11}} + C.$$

$$104. \int \frac{dx}{x^2+3x+1}. \text{ Resp.: } \frac{1}{\sqrt{5}} \ln \left| \frac{2x+3-\sqrt{5}}{2x+3+\sqrt{5}} \right| + C.$$

$$105. \int \frac{dx}{x^2-6x+5}. \text{ Resp.: } \frac{1}{4} \ln \left| \frac{x-5}{x-1} \right| + C.$$

106. $\int \frac{dx}{3x^2 - 2x + 2}$. Resp.: $\frac{1}{\sqrt{5}} \operatorname{arctg} \frac{3x-1}{\sqrt{5}} + C$.
107. $\int \frac{(6x-7)dx}{3x^2 - 7x + 11}$. Resp.: $\ln|3x^2 - 7x + 11| + C$.
108. $\int \frac{(3x-2)dx}{5x^2 - 3x + 2}$. Resp.: $\frac{3}{10} \ln(5x^2 - 3x + 2) - \frac{11}{5\sqrt{31}} \operatorname{artg} \frac{10x-3}{\sqrt{31}} + C$.
109. $\int \frac{3x-1}{x^2 - x + 1} dx$. Resp.: $\frac{3}{2} \ln(x^2 - x + 1) + \frac{1}{\sqrt{3}} \operatorname{artg} \frac{2x-1}{\sqrt{3}} + C$.
110. $\int \frac{7x+1}{6x^2 + x - 1} dx$. Resp.: $\frac{2}{3} \ln(3x+1) + \frac{1}{2} \ln(2x+1) + C$.
111. $\int \frac{2x-1}{5x^2 - x + 2} dx$. Resp.: $\frac{1}{5} \ln(5x^2 - x + 2) + \frac{8}{5\sqrt{39}} \operatorname{arctg} \frac{10x-1}{\sqrt{39}} + C$.
112. $\int \frac{6x^4 - 5x^3 + 4x^2}{2x^2 - x + 1} dx$. Resp.: $x^3 - \frac{x^2}{2} + \frac{1}{4} \ln|2x^2 - x + 1| + \frac{1}{2\sqrt{7}} \operatorname{arctg} \frac{4x-1}{\sqrt{7}} + C$.
113. $\int \frac{dx}{2\cos^2 x + \operatorname{sen} x \cos x + \operatorname{sen}^2 x}$. Resp.: $\frac{2}{\sqrt{7}} \operatorname{acrtg} \frac{2\operatorname{tg} x + 1}{\sqrt{7}} + C$.

Integrales del tipo $\int \frac{Ax+B}{\sqrt{ax^2+bx+C}} dx$:

114. $\int \frac{dx}{\sqrt{-4x^2 - 3x + 2}}$. Resp.: $\frac{1}{2} \operatorname{arcsen} \frac{8x+3}{\sqrt{41}} + C$.
115. $\int \frac{dx}{\sqrt{x^2 + x + 1}}$. Resp.: $\ln \left| x + \frac{1}{2} + \sqrt{x^2 + x + 1} \right| + C$.
116. $\int \frac{dS}{\sqrt{S^2 + 2aS}}$. Resp.: $\ln \left| S + a + \sqrt{2aS + S^2} \right| + C$.
117. $\int \frac{dx}{\sqrt{-3x^2 - 7x + 5}}$. Resp.: $\frac{1}{\sqrt{3}} \operatorname{arcsen} \frac{6x+7}{\sqrt{109}} + C$.
118. $\int \frac{dx}{\sqrt{x(3x+5)}}$. Resp.: $\frac{1}{\sqrt{3}} \ln|6x+5 + \sqrt{12x(3x+5)}| + C$.
119. $\int \frac{dx}{\sqrt{-x^2 - 3x + 2}}$. Resp.: $\operatorname{arsen} \frac{2x+3}{\sqrt{17}} + C$.

120. $\int \frac{dx}{\sqrt{5x^2 - x - 1}}$. Resp.: $\frac{1}{\sqrt{5}} \ln|10x - 1 + \sqrt{20(5x^2 - x - 1)}| + C$.
121. $\int \frac{2ax + b}{\sqrt{ax^2 + bx + c}} dx$. Resp.: $2\sqrt{ax^2 + bx + c} + C$.
122. $\int \frac{(x + 3)dx}{\sqrt{4x^2 + 4x + 3}}$. Resp.: $\frac{1}{4}\sqrt{4x^2 + 4x + 3} + \frac{5}{4}\ln|2x + 1 + \sqrt{4x^2 + 4x + 3}| + C$.
123. $\int \frac{(x - 3)dx}{\sqrt{-11x^2 + 66x + 3}}$. Resp.: $\frac{-1}{11}\sqrt{3 + 66x - 11x^2} + C$.
124. $\int \frac{(x + 3)dx}{\sqrt{-4x^2 + 4x + 3}}$. Resp.: $-\frac{1}{4}\sqrt{3 + 4x - 4x^2} + \frac{7}{4}\arcsen\frac{2x - 1}{2} + C$.
125. $\int \frac{(3x + 5)dx}{\sqrt{x(2x - 1)}}$. Resp.: $\frac{3}{2}\sqrt{2x^2 - x} + \frac{23}{4\sqrt{2}}\ln(4x - 1 + \sqrt{8(2x^2 - x)}) + C$.

II. Integración por partes:

126. $\int xe^x dx$. Resp.: $e^x(x - 1) + C$.
127. $\int x \ln x dx$. Resp.: $\frac{1}{2}x^2 \left(\ln x - \frac{1}{2} \right) + C$.
128. $\int x \sen x dx$. Resp.: $\sen x - x \cos x + C$.
129. $\int \ln x dx$. Resp.: $x(\ln x - 1) + C$.
130. $\int \arcsen x dx$. Resp.: $x \arcsen x + \sqrt{1 - x^2} + C$.
131. $\int \ln(1 - x) dx$. Resp.: $-x - (1 - x)\ln(1 - x) + C$.
132. $\int x^n \ln x dx$. Resp.: $\frac{x^{n+1}}{n+1} \left(\ln x - \frac{1}{n+1} \right) + C$.
133. $\int x \arctg x dx$. Resp.: $\frac{1}{2}[(x^2 + 1)\arctg x - x] + C$.
134. $\int x \arcsen x dx$. Resp.: $\frac{1}{4}[(2x^2 - 1)\arcsen x + x\sqrt{1 - x^2}] + C$.

135. $\int (\ln(x^2 + 1))dx$. Resp.: $x\ln(x + 1) - 2x + 2\arctg x + C$.
136. $\int \arctg \sqrt{x} dx$. Resp.: $(x + 1)\arctg \sqrt{x} - \sqrt{x} + C$.
137. $\int \frac{\arcsen \sqrt{x}}{\sqrt{x}} dx$. Resp.: $2\sqrt{x}\arcsen \sqrt{x} + 2\sqrt{1-x} + C$.
138. $\int \arcsen \sqrt{\frac{x}{x+1}} dx$. Resp.: $x\arcsen \sqrt{\frac{x}{x+1}} - \sqrt{x} + \arctg \sqrt{x} + C$.
139. $\int x \cos^2 x dx$. Resp.: $\frac{x^2}{4} + \frac{1}{4}x \sin 2x + \frac{1}{8} \cos 2x + C$.
140. $\int \frac{x \arcsen x}{\sqrt{1-x^2}} dx$. Resp.: $x - \sqrt{1-x^2} \arcsen x + C$.
141. $\int \frac{x \arctg x}{(x^2 + 1)^2} dx$. Resp.: $\frac{x}{4(1+x^2)} + \frac{1}{4} \arctg x - \frac{1}{2} \frac{\arctg x}{1+x^2} + C$.
142. $\int x \arctg \sqrt{x^2 - 1} dx$. Resp.: $\frac{1}{2} x^2 \arctg \sqrt{x^2 - 1} - \frac{1}{2} \sqrt{x^2 - 1} + C$.
143. $\int \frac{\arcsen x}{x^2} dx$. Resp.: $\ln \left| \frac{1 - \sqrt{1-x^2}}{x} \right| - \frac{1}{x} \arcsen x + C$.
144. $\int \ln(x + \sqrt{1+x^2}) dx$. Resp.: $x \ln |x + \sqrt{1+x^2}| - \sqrt{1+x^2} + C$.
145. $\int \arcsen x \frac{xdx}{\sqrt{(1-x^2)^3}}$. Resp.: $\frac{\arcsen x}{\sqrt{1-x^2}} + \frac{1}{2} \ln \left| \frac{1-x}{1+x} \right| + C$.

Utilizar sustituciones trigonométricas en los ejemplos siguientes:

146. $\int \frac{\sqrt{a^2 - x^2}}{x^2} dx$. Resp.: $-\frac{\sqrt{a^2 - x^2}}{x} - \arcsen \frac{x}{a} + C$.
147. $\int x^2 \sqrt{4 - x^2} dx$. Resp.: $2\arcsen \frac{x}{2} - \frac{1}{2} x \sqrt{4 - x^2} + \frac{1}{4} x^3 \sqrt{4 - x^2} + C$.
148. $\int \frac{dx}{x^2 \sqrt{1+x^2}}$. Resp.: $-\frac{\sqrt{1+x^2}}{x} + C$.

$$149. \int \frac{\sqrt{x^2 + a^2}}{x} dx. \text{ Resp.: } \sqrt{x^2 - a^2} - a \arccos \frac{a}{x} + C.$$

$$150. \int \frac{dx}{(\sqrt{a^2 + x^2})^3}. \text{ Resp.: } \frac{x}{a^2} \frac{1}{\sqrt{a^2 + x^2}} + C.$$

Integración de funciones racionales:

$$151. \int \frac{2x - 1}{(x - 1)(x - 2)} dx. \text{ Resp.: } \ln \left| \frac{(x - 2)^3}{x - 1} \right| + C.$$

$$152. \int \frac{x dx}{(x + 1)(x + 3)(x + 5)}. \text{ Resp.: } \frac{1}{8} \ln \frac{(x + 3)^6}{(x + 5)^5(x + 1)} + C.$$

$$153. \int \frac{x^5 + x^4 - 8}{x^3 - 4x} dx. \text{ Resp.: } \frac{x^3}{3} + \frac{x^2}{2} + 4x + \ln \left| \frac{x^2(x - 2)^5}{(x + 2)^3} \right| + C.$$

$$154. \int \frac{x^4 dx}{(x^2 - 1)(x + 2)}. \text{ Resp.: } \frac{x^2}{2} - 2x + \frac{1}{6} \ln \frac{(x - 1)}{(x + 1)^3} + \frac{16}{3} \ln(x + 2) + C.$$

$$155. \int \frac{(x - 8) dx}{x^3 - 4x^2 + 4x}. \text{ Resp.: } \frac{3}{x - 2} + \ln \frac{(x - 2)^2}{x^2} + C.$$

$$156. \int \frac{(3x + 2) dx}{x(x + 1)^3}. \text{ Resp.: } \frac{4x + 3}{2(x + 1)^2} + \ln \frac{x^2}{(x + 1)^2} + C.$$

$$157. \int \frac{x^2 dx}{(x + 2)^2(x + 4)^2}. \text{ Resp.: } -\frac{5x + 12}{x^2 + 6x + 8} + \ln \left(\frac{x + 4}{x + 2} \right)^2 + C.$$

$$158. \int \frac{dx}{x(x^2 + 1)}. \text{ Resp.: } \ln \frac{x}{\sqrt{x^2 + 1}} + C.$$

$$159. \int \frac{2x^2 - 3x - 3}{(x - 1)(x^2 - 2x + 5)} dx. \text{ Resp.: } \ln \frac{(x^2 - 2x + 5)^{\frac{3}{2}}}{x - 1} + \frac{1}{2} \arctg \frac{x - 1}{2} + C.$$

$$160. \int \frac{x^3 - 6}{x^4 + 6x^2 + 8} dx. \text{ Resp.: } \ln \frac{x^2 + 4}{\sqrt{x^2 + 2}} + \frac{3}{2} \arctg \frac{x}{2} - \frac{3}{\sqrt{2}} \arctg \frac{x}{\sqrt{2}} + C.$$

$$161. \int \frac{2x^2 - 3x - 3}{(x - 1)(x^2 - 2x - 5)} dx. \text{ Resp.: } \ln \frac{(x^2 - 2x - 5)^{\frac{3}{2}}}{(x - 1)} + \frac{1}{2} \arctg \frac{x - 1}{2} + C.$$

$$162. \int \frac{dx}{x^3 + 1}. \text{ Resp.: } \frac{1}{6} \ln \frac{(x + 1)^2}{x^2 - x + 1} + \frac{1}{\sqrt{3}} \arctg \frac{2x - 1}{\sqrt{3}} + C.$$

$$\begin{aligned}
163. \int \frac{3x-7}{x^3+x^2+4x+4} dx. \text{ Resp.: } \ln \frac{x^2+4}{(x+1)^2} + \frac{1}{2} \operatorname{arctg} \frac{x}{2} + C. \\
164. \int \frac{4 dx}{x^4+1}. \text{ Resp.: } \frac{1}{\sqrt{2}} \ln \frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1} + \sqrt{2} \operatorname{arctg} \frac{\sqrt{2}x}{1-x^2} + C. \\
165. \int \frac{(4x^2-8x)dx}{(x-1)^2(x^2+1)^2}. \text{ Resp.: } \frac{3x^2-1}{(x-1)(x^2+1)} + \ln \frac{(x-1)^2}{x^2+1} + \operatorname{arctg} x + C. \\
166. \int \frac{dx}{(x^2-x)(x^2-x-1)^2}. \text{ Resp.: } \ln \frac{x-1}{x} - \frac{10}{3\sqrt{3}} \operatorname{arctg} \frac{2x-1}{\sqrt{3}} - \\
- \frac{2x-1}{3(x^2-x-1)} + C.
\end{aligned}$$

Integración de funciones irracionales:

$$\begin{aligned}
167. \int \frac{\sqrt{x}}{\sqrt[4]{x^3}+1} dx \text{ Resp.: } \frac{4}{3} \left[\sqrt[4]{x^3} - \ln \left(\sqrt[4]{x^3} + 1 \right) \right] + C \\
168. \int \frac{\sqrt{x^3} - \sqrt[3]{x}}{6\sqrt[4]{x}} dx \text{ Resp.: } \frac{2}{27} \sqrt[4]{x^9} - \frac{2}{13} \sqrt[12]{x^{13}} + C \\
169. \int \sqrt{\frac{1-x}{1+x}} \frac{dx}{x^2} \text{ Resp.: } \ln \left| \frac{\sqrt{1-x} + \sqrt{1+x}}{\sqrt{1-x} - \sqrt{1+x}} \right| - \frac{\sqrt{1-x^2}}{x} + C \\
170. \int \sqrt{\frac{1-x}{1+x}} \frac{dx}{x} \text{ Resp.: } \ln \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} + 2 \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} + C \\
171. \int \frac{\sqrt[7]{x} + \sqrt{x}}{\sqrt[7]{x^8} + \sqrt[14]{x^{15}}} dx \text{ Resp.: } 14 \left[\sqrt[14]{x} - \frac{1}{2} \sqrt[7]{x} + \frac{1}{3} \sqrt[14]{x^3} - \frac{1}{4} \sqrt[7]{x^2} + \frac{1}{5} \sqrt[14]{x^5} \right] + C. \\
172. \int \sqrt{\frac{2+3x}{x-3}} dx. \text{ Resp.: } \sqrt{3x^2-7x-6} + \frac{11}{2\sqrt{3}} \ln \left(x - \frac{7}{6} + \sqrt{x^2 - \frac{7}{3}x - 2} \right) + C.
\end{aligned}$$

Integrales del tipo $\int R(x, \sqrt{ax^2+bx+c}) dx$

$$\begin{aligned}
173. \int \frac{dx}{x\sqrt{x^2-x+3}}. \text{ Resp.: } \frac{1}{\sqrt{3}} \ln \left| \frac{\sqrt{x^2-x+3} - \sqrt{3}}{x} + \frac{1}{2\sqrt{3}} \right| + C. \\
174. \int \frac{dx}{x\sqrt{-x^2+x+2}}. \text{ Resp.: } \frac{-1}{\sqrt{2}} \ln \left| \frac{\sqrt{-x^2+x+2} + \sqrt{2}}{x} + \frac{1}{2\sqrt{2}} \right| + C.
\end{aligned}$$

175. $\int \frac{dx}{x\sqrt{x^2+4x-4}}$. Resp.: $\frac{1}{2}\arcsen\frac{x-2}{x\sqrt{2}}+C$.
176. $\int \frac{\sqrt{x^2+2x}}{x}dx$. Resp.: $\sqrt{x^2+2x}+\ln|x+1+\sqrt{x^2+2x}|+C$.
177. $\int \frac{dx}{\sqrt{(2x-x^2)^3}}$. Resp.: $\frac{x-1}{\sqrt{2x-x^2}}+C$.
178. $\int \sqrt{2x-x^2}dx$. Resp.: $\frac{1}{2}[(x-1)\sqrt{2x-x^2}+\arcsen(x-1)]+C$.
179. $\int \frac{dx}{x-\sqrt{x^2-1}}$. Resp.: $\frac{x^2}{2}+\frac{x}{2}\sqrt{x^2-1}-\frac{1}{2}\ln|x+\sqrt{x^2-1}|+C$.
180. $\int \frac{dx}{(1+x)\sqrt{1+x+x^2}}$. Resp.: $\ln\left|\frac{x+\sqrt{1+x+x^2}}{2+x+\sqrt{1+x+x^2}}\right|+C$.
181. $\int \frac{x+1}{(2x+x^2)\sqrt{2x+x^2}}dx$. Resp.: $\frac{-1}{\sqrt{2x+x^2}}+C$.
182. $\int \frac{1-\sqrt{1+x+x^2}}{x\sqrt{1+x+x^2}}dx$. Resp.: $\ln\left|\frac{2+x-2\sqrt{1+x+x^2}}{x^2}\right|+C$.
183. $\int \frac{\sqrt{x^2+4x}}{x^2}dx$. Resp.: $-\frac{8}{x+\sqrt{x^2+4x}}+\ln|x+2+\sqrt{x^2+4x}|+C$.

Integración de los binomios diferenciales:

184. $\int \frac{\sqrt{1+\sqrt[3]{x}}}{\sqrt[3]{x^2}}dx$. Resp.: $2\left(1+x^{\frac{1}{3}}\right)^{\frac{3}{2}}+C$.
185. $\int x^{\frac{1}{3}}\left(2+x^{\frac{2}{3}}\right)^{\frac{1}{4}}dx$. Resp.: $\frac{10x^{\frac{2}{3}}-16}{15}\left(2+x^{\frac{2}{3}}\right)^{\frac{5}{4}}+C$.
186. $\int \frac{dx}{(1+x^2)^{\frac{3}{2}}}$. Resp.: $\frac{x}{\sqrt{1+x^2}}+C$.
187. $\int \frac{dx}{x^2(1+x^2)^{\frac{3}{2}}}$. Resp.: $-(1+x^2)^{-\frac{1}{2}}\left(2x+\frac{1}{x}\right)+C$.
188. $\int \sqrt[4]{\left(1+x^{\frac{1}{2}}\right)^3}dx$. Resp.: $\frac{8}{77}(7\sqrt{x}-4)(1+\sqrt{x})^{\frac{7}{4}}+C$.

$$189. \int \frac{\sqrt{2 - \sqrt[3]{x}}}{\sqrt[3]{x}} dx. \text{ Resp.: } \frac{2(4 + 3\sqrt[3]{x})(2 - \sqrt[3]{x})^{\frac{3}{2}}}{5} + C.$$

$$190. \int x^5 \sqrt[3]{(1 + x^3)^2} dx. \text{ Resp.: } \frac{5x^3 - 3}{40} (1 + x^3)^{\frac{5}{3}} + C.$$

Integración de las funciones trigonométricas:

$$191. \int \operatorname{sen}^3 x dx. \text{ Resp.: } \frac{1}{3} \cos^3 x - \cos x + C.$$

$$192. \int \operatorname{sen}^5 x dx. \text{ Resp.: } -\cos x + \frac{2}{3} \cos^3 x - \frac{\cos^5 x}{5} + C.$$

$$193. \int \cos^4 x \operatorname{sen}^3 x dx. \text{ Resp.: } -\frac{1}{5} \cos^5 x + \frac{1}{7} \cos^7 x + C.$$

$$194. \int \frac{\cos^3 x}{\operatorname{sen}^4 x} dx. \text{ Resp.: } \csc x - \frac{1}{3} \csc^3 x + C.$$

$$195. \int \cos^2 x dx. \text{ Resp.: } \frac{x}{2} + \frac{1}{4} \operatorname{sen} 2x + C.$$

$$196. \int \operatorname{sen}^4 x dx. \text{ Resp.: } \frac{3}{8} x - \frac{\operatorname{sen} 2x}{4} + \frac{\operatorname{sen} 4x}{32} + C.$$

$$197. \int \cos^6 x dx. \text{ Resp.: } \frac{1}{16} (5x + 4 \operatorname{sen} 2x - \frac{\operatorname{sen}^3 2x}{3} + \frac{3}{4} \operatorname{sen} 4x) + C.$$

$$198. \int \operatorname{sen}^4 x \cos^4 x dx. \text{ Resp.: } \frac{1}{128} \left(3x - \operatorname{sen} 4x + \frac{\operatorname{sen} 8x}{8} \right) + C.$$

$$199. \int \operatorname{tg}^3 x dx. \text{ Resp.: } \frac{\operatorname{tg}^2 x}{2} + \ln |\cos x| + C.$$

$$200. \int \operatorname{cotg}^5 x dx. \text{ Resp.: } -\frac{1}{4} \operatorname{cotg}^4 x + \frac{1}{2} \operatorname{cotg}^2 x + \ln |\operatorname{sen} x| + C.$$

$$201. \int \operatorname{cotg}^3 x dx. \text{ Resp.: } -\frac{\operatorname{cotg}^2 x}{2} - \ln |\operatorname{sen} x| + C.$$

$$202. \int \operatorname{sc}^3 x dx. \text{ Resp.: } \frac{\operatorname{tg}^7 x}{7} + \frac{3 \operatorname{tg}^5 x}{5} + \operatorname{tg}^3 x + \operatorname{tg} x + C.$$

$$203. \int \operatorname{tg}^4 x \operatorname{sc}^4 x dx. \text{ Resp.: } \frac{\operatorname{tg}^7 x}{7} + \frac{\operatorname{tg}^5 x}{5} + C.$$

204. $\int \frac{dx}{\cos^4 x}$. Resp.: $\operatorname{tg} x + \frac{1}{3} \operatorname{tg}^3 x + C$.
205. $\int \frac{\cos x}{\operatorname{sen}^2 x} dx$. Resp.: $-\operatorname{csc} x + C$.
206. $\int \frac{\operatorname{sen}^3 x}{\sqrt[3]{\cos^4 x}} dx$. Resp.: $\frac{3}{5} \cos^{\frac{5}{3}} x + 3 \cos^{-\frac{1}{3}} x + C$.
207. $\int \operatorname{sen} x \operatorname{sen} 3x dx$. Resp.: $-\frac{\operatorname{sen} 4x}{8} + \frac{\operatorname{sen} 2x}{4} + C$.
208. $\int \cos 4x \cos 7x dx$. Resp.: $\frac{\operatorname{sen} 11x}{22} + \frac{\operatorname{sen} 3x}{6} + C$.
209. $\int \cos 2x \operatorname{sen} 4x dx$. Resp.: $-\frac{\cos 6x}{12} - \frac{\cos 2x}{4} + C$.
210. $\int \operatorname{sen} \frac{1}{4} x \cos \frac{3}{4} x dx$. Resp.: $-\frac{\cos x}{2} + \cos \frac{1}{2} x + C$.
211. $\int \frac{dx}{4 - 5 \operatorname{sen} x}$. Resp.: $\frac{1}{3} \ln \left| \frac{\operatorname{tg} \frac{x}{2} - 2}{2 \operatorname{tg} \frac{x}{2} - 1} \right| + C$.
212. $\int \frac{dx}{5 - 3 \cos x}$. Resp.: $\frac{1}{2} \operatorname{arctg} \left| 2 \operatorname{tg} \frac{x}{2} \right| + C$.
213. $\int \frac{\operatorname{sen} x dx}{1 + \operatorname{sen} x}$. Resp.: $\frac{2}{1 + \operatorname{tg} \frac{x}{2}} + x + C$.
214. $\int \frac{\cos x dx}{1 + \cos x}$. Resp.: $x - \operatorname{tg} \frac{x}{2} + C$.
215. $\int \frac{\operatorname{sen} 2x}{\cos^4 x + \operatorname{sen}^4 x} dx$. Resp.: $\operatorname{arctg}(2 \operatorname{sen}^2 x - 1) + C$.
216. $\int \frac{dx}{(1 + \cos x)^2}$. Resp.: $\frac{1}{2} \operatorname{tg} \frac{x}{2} + \frac{1}{6} \operatorname{tg}^3 \frac{x}{2} + C$.
217. $\int \frac{dx}{\operatorname{sen}^2 x + \operatorname{tg}^2 x}$. Resp.: $-\frac{1}{2} [\cot g x + \frac{1}{\sqrt{2}} \operatorname{arctg} \left(\frac{\operatorname{tg} x}{\sqrt{2}} \right)] + C$.
218. $\int \frac{\operatorname{sen}^2 x}{1 + \cos^2 x} dx$. Resp.: $\sqrt{2} \operatorname{arctg} \left(\frac{\operatorname{tg} x}{\sqrt{2}} \right) - x + C$.

II. Calcular las siguientes integrales:

219. $\int \frac{dx}{x\sqrt{1-(\ln x)^2}}$. Resp.: $\arcsen(\ln x) + C$.
220. $\int \frac{dx}{\sqrt{(1+x^2)\ln(x+\sqrt{1+x^2})}}$. Resp.: $2\sqrt{\ln(x+\sqrt{1+x^2})} + C$.
221. $\int (4+2x+x^2)e^{-2x} dx$. Resp.: $-\frac{1}{4}(2x^2+6x+11)e^{-2x} + C$.
222. $\int x \sec^2 ax dx$. Resp.: $\frac{1}{a} x \tan ax + \frac{1}{a^2} \ln(\cos ax) + C$.
223. Calcular una fórmula de reducción para la integral $\int \frac{x^n}{\sqrt{a^2-x^2}} dx$.
Como aplicación, calcular $\int \frac{x^4}{\sqrt{a^2-x^2}} + C$.
224. Calcular una fórmula de reducción para la integral $I_n = \int \frac{dx}{(a^2+x^2)^n}$.
Como aplicación, calcular I_3 .
225. Calcular una fórmula de reducción para la integral $I_n = \int \tan^n x dx$.
Como aplicación, calcular I_5 .
226. Calcular $\int \frac{x^5-x^4-3x+5}{x^4-2x^3+2x^2-2x+1} dx$. $I = \frac{x^2}{2} + x - \frac{1}{x-1} + \ln(x^2+1) - 2\ln(x-1) + \arctan x + C$.
227. Calcular
$$\int \frac{x^3+x^2+x+1}{(x-3)\sqrt{x-2}\sqrt{x^3-x^2-5x-3}} dx$$

Resp.: $I = \sqrt{x^2-5x+6} + \frac{11}{2} \operatorname{Argch}(2x-5) - 20\sqrt{\frac{x-2}{x-3}} + C$.
228. Calcular
$$I = \int \cos 2x \sin 6x dx$$

Resp.: $I = -\frac{1}{16}(\cos 8x + 2\cos 4x) + C$.

229. Calcular

$$I = \int \operatorname{sen}^4 x \cos^4 x dx$$

$$\text{Resp.: } I = \frac{1}{64} \left(\frac{3}{2}x - \frac{1}{2}\operatorname{sen}4x + \frac{1}{16}\operatorname{sen}8x \right) + C.$$